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AUTHOR Hendrickson, A. Dean
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ABSTRACT

In the fall of 1980, a course titled Developing Critical Thinking Skills was approved at the University of Minnesota-Duluth for offering as one of the options in the liberal education category called Communication, Language, and Symbolic Systems. This paper provides a description of the course (designed to provide experiences with methods and materials diverse enough to appeal to the needs of the different conceptual styles students have), the outline of course topics, and strategies used to teach the topics on the outline. Major areas are: the nature of critical thinking; the nature of thinking and intelligence; logical (if-then) reasoning; combinatorial reasoning; orderings and patterns; comparing and classifying; variation and rates of variation; analogies; and problem-solving. The first two topics are treated through lecture and discussion. The remainder of the course is a systematic involvement with each kind of reasoning process. Initial experience is given with concrete materials or game activity whenever suitable; symbolic representations and more abstract situations are gradually introduced. Students are assigned to groups of four or five for activities (which are described) and are given opportunities to serve both as a learner and a tutor within the group. (JN)

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DEVELOPING CRITICAL THINKING SKILLS

A.Dean Hendrickson

University of Minnesota Duluth

The work of Jean Piaget first drew the attention of educators involved with early childhood education and elementary education. Several groups utilized the stage development theory of Piaget in designing the sequence of activities and in the emphasis on classification, ordering, and the recognition of relations. Some of the better known examples of these curricula include the Science Curriculum Improvement Study, the High Scope early childhood program, the Elementary Science Study, and the Nuffield projects in science and mathematics in England.

Piaget and Inhelder(1958) made the first major contribution to developing a focus on formal operational reasoning patterns. Soon after that several researchers in the Geneva school, in England and a few in North America began to determine the extent to which students age twelve and up were able to use proportional reasoning, combinatorial reasoning, the control and manipulation of variables, and analogy in thinking. Some of these investigators found surprisingly high numbers of students in secondary schools and in colleges who were still primarily concrete operational in their performance on a variety of tasks that required formal operational thought processes. Many of these results indicated that a full use of formal reasoning develops at a much later age than a strictly Piagetian description would indicate. Results of these investigators used in the development of the course that is the subject of this paper include those by Lovell(1969,1971), Karplus, et. al.(1970,1974,1977,1978,1979), Arlin(1976), and Renner, et. al.(1976). Renner and Stafford (1976) concluded

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that three-fourths of the students in secondary schools are primarily concrete operational in their thinking, and that instruction in science would have to be modified to take this into proper account. Populations of students in their first two years of college have performed at only a slightly higher level. McKinnon(1970) found tht over fifty percent of the college students in a sample taken from seven colleges of different kinds that he tested gave concrete operational responses to tasks involving proportional and combinatorial reasoning. He then designed a course in science that emphasized these processes and found it was possible to accelerate the progress of college students into formal thought.

In response to these and other studies of college students, in 1977, the Lawrence Hall of Science group developed a series of workshops for secondary and college teachers in the sciences. In these they pointed where, in the different content areas in science, formal operations were needed and suggested ways to initiate topics to help to develop this kind of thinking, rather than tacitly to assume that all students already were capable of it. Lochhead and Clement(1979) edited a summary of research on thinking skills to which these investigators and others equally concerned with this problem contributed. In the fall of 1980 the course entitled Developing Critical Thinking Skills was approved at the University of Minnesota Duluth for offering as one of the options in the liberal education category called Communication, Language and Symbolic Systems. Since that time, two sections of the course enrolling 35 students in each section have been taught during each quarter, or three times in an academic year. After two years the number of credit hours was increased from three to four when it was found that more time was needed to give students enough experience with each kind of thinking process.

The topical outline for this class is:

I The Nature of Critical Thinking

A. What is critical thinking?

B. Some examples of critical thinking that are commonly found in solving everyday problems.

C. The need for critical thinking skills in different content areas in the university.

II The Nature of Thinking and Intelligence

A. What is intelligence?

B. What is thinking?

C. What kinds of reasoning are associated with mature thinking?

III Logical(if-then)Reasoning

A. Conjunction.

B. Disjunction.

C. Negation.

D. Implication.

E. De Morgan's laws.

F. Truth tables and their use.

G. Errors common in implicational reasoning.

IV Combinatorial Reasoning

A. Making selections from available choices.

B. Multiple selections and the multiplication principle.

C. Combined selections and the ordering of selections.

D. The effect of imposing conditions or requirements on the choices available.

V Orderings and Patterns

A. Permutations within groups.

B. Permutations vs. combinations.

C. Pattern analysis, identification and completion.

D. Sequences and series.

E. Comparisons within and between orderings.

VI Comparing and classifying

A. Attributes and values of attributes.

B. How classification is affected by conditions imposed upon membership.

C. The use of charts, diagrams and tables in classifying.

D. How combinatorial and implication reasoning interact with classification.

VII Variation and Rates of Variation

A. Constant rate of change and examples.

B. Ratio, proportion and proportional reasoning.

C. Inverse rates of change.

D. Variable rates of change.

E. Limits and limiting processes.

F. Manipulation and control of variables.

VIII Analogies

A. The kinds of relationships expressible as analogies.

B. Analyzing analogies for relationships.

C. Completing analogies.

D. Creating analogies.

IX Problem Solving

A. The nature of problem solving.

B. Problem solving strategies.

C. Problem solving heuristics and helps.

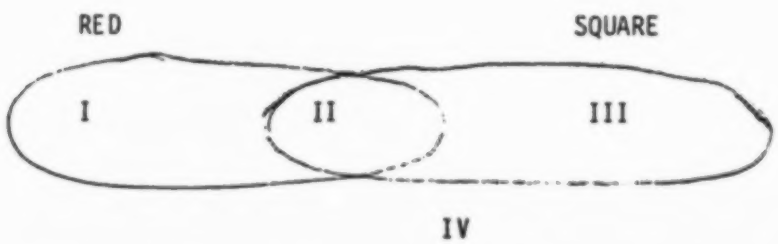
D. Using big ideas in problem solving.

In the five years that this class has been taught, it has undergone several changes in addition to the increase in credit. Problem Solving and Comprehension, Whimbey and Lochhead, was used as a text for the first two years. However this text deals with only half of the topics in this course and it is no longer a text, although it is still a good resource. Three years ago a student guide was developed that is currently being used. It is now subject to extensive revision. This guide has brief introductions to the topics, several sample problems with solutions for some, assignments to be completed and handed in, descriptions of several common fallacies in thinking, and a set of questions to be answered upon completion of the viewing of five of the films in the series, Search For Solutions. The classroom activities have been developed using a variety of sources, including several problem books in thought and logic, problems taken from magazines and journals, and several commercial products. Many of the most-used resources are listed in the appendix. Two examinations are given during the term. The mid-term test is a set of 15 problems that require the thinking processes emphasized to that point. The final examination is a set of 30-35 problems requiring all processes emphasized in the course. A sample of each kind of problem used is found in the appendix.

The first two topics on the nature of thinking and intellectual functioning are treated through lecture and discussion. The remainder of the course is a systematic involvement with each kind of reasoning process. Initial experience is given with concrete materials or game activity wherever suitable. Symbolic representations and more abstract situations are gradually introduced. Students

are assigned to groups of four and five for all activity and are encouraged to study within these groups outside of class as well. Each student is assigned responsibility for the learning of ALL members of the group and verbal interaction within the groups is encouraged. Every attempt is made to give each student the opportunity to serve as both a learner and a tutor within the group.

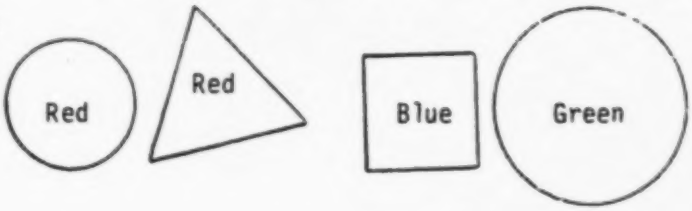
Involvement with logical reasoning is introduced with activities that require the use of attribute materials. The particular materials used are those finalized by the Elementary Science Study development group. These are blocks of four colors, four different shapes and in two distinctly different sizes. The students play similarity and difference games of various kinds to become familiar with the attributes and values of the attributes of these blocks. The same materials are then used to focus on the meaning of NOT, AND and OR. As an example of one activity used for this purpose, students are instructed to collect all squares within one looped cord and all red blocks inside a second looped cord. They must realize that the loops must be arranged as in the diagram below to account for the joint presence of squareness and redness in some blocks.



Then students are asked to identify and describe the properties of the blocks sorted out into the regions labelled I, II, III and IV, using AND, OR and NOT. Those in each region are to be described using several different sentences.

Particular attention is paid to describing IV using NOT and describing those that are NOT II to develop the De Morgan laws. Particular attention is given the difference between AND/OR - or inclusive or - and EITHER-OR, or exclusive or. It is surprising that college students have so much difficulty with the use of OR and with the use of NOT. In this kind of activity, students are urged to use Venn diagrams to keep their information organized, and several problems for which Venn diagrams are almost necessary are solved in class.

During this activity the truth tables for AND, for AND/OR for EITHER-OR and for NOT are developed. Truth tables are used to show the validity of the De Morgan laws. Collections of transparent versions of the attribute materials are placed on the overhead projector and students are asked to generate as many statements in if-then form that are true of these collections as they can. An example of such a collection with a few typical statements that might be produced is:



If Green, then Circle If Square, then Blue

From the list of statements generated for a particular collection, one is chosen and the students are asked to suggest pieces to add or subtract that would make the statement false. Then they are asked to do this so that the IF part would be false while the THEN part remains true, They then are asked to

make both the IF part and the THEN part false. They quickly see that the only way that one can guarantee the statement has become false is to keep the IF part true while making the THEN part false. The basic truth table for implication is then developed and a considerable amount of time spent in analyzing IF-THEN statements with combinations of AND, OR and NOT in one or other of the antecedent and consequent or in both. The meaning of necessity and sufficiency is developed and time is spent on the difference between IF and ONLY IF. Time is also spent on the meaning of ALL and SOME and the negations of these. The role of the contrapositive in proof and the problems associated with confusing the converse with the original implication are discussed in detail and several examples are given. Three assignments that deal with this topic are made. These require students to convert statements from forms that use IF, ONLY IF, SUFFICIENT, NECESSARY, ALL, SOME, AND, OR and NOT into other forms using these terms. Truth tables are used to solve detective stories, riddles, and to answer questions about conclusions that can be drawn about the conditions associated with two or more conditional statements that are known to be true. Several of the materials published by Midwest Publications and listed among the Resources are used in this activity. Particular attention is given to the chaining of several if-then statements and then drawing conclusions in if-then form that are true within this chain.

Combinatorial reasoning is introduced using cubes of six different colors. Initially students are given several cubes of two different colors and asked to find one example, using one cube of a particular color, of the collections that have one color present and two colors present. This result is recorded in the first row of the table below. Students are asked to complete the remaining rows on the table. This will reveal all possible combinations of

colors taken two,three,...,six at a time that can be made from, in turn, one,two,..., six available colors from which to choose. The reader will recognize the Pascal triangle in this table.

NUMBER OF COLORS AVAILABLE	NUMBER OF COLORS PRESENT IN SETS OF EACH SIZE								TOTAL
	0	1	2	3	4	5	6		
ONE									
TWO									
THREE									
FOUR									
FIVE									
SIX									

The following factors about the resulting table are emphasized:

How previously formed collections are still possible given another choice of color.

How new collections are made from the previously formed collections when another color is made available.

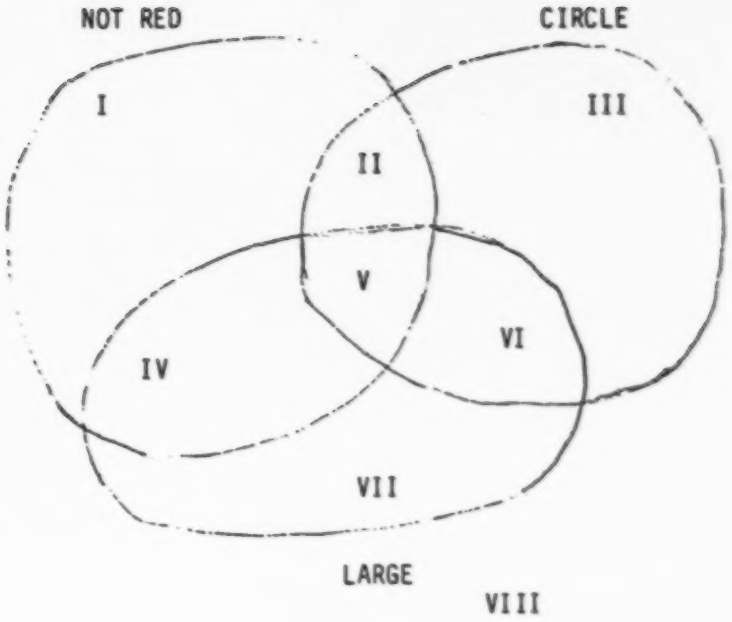
Where the "knight's move" comes from and how to use it to continue the table.

How the sum of the entries in each row is related to the binary decision making process of choosing to put something into a collection, or to leave it out of the collection.

The relationships developed from analysis of the table are generalized to the process of making choices of any kind from available options.

The nature of an ordered selection process in which a first choice is made, followed by a second choice, etc. is considered, and the fact that this results in all possible orderings of the collections thus chosen is brought out with examples. Students are then asked to establish linear orderings of collections of cubes of successively two, three,.., six different colors. The difference between the ordering process, or considering permutations, and the selection process, or forming combinations, is emphasized with several examples and problems. The problems involve selecting from menus, forming committees, the possible starting lineups for basketball and hockey squads, the number of experiments needed to determine how different numbers of reagents might react and similar situations. These are solved within groups, as a class, and in completing of individual assignments.

The attribute materials are once again used to introduce the idea of meeting a third condition for membership in classes. After sorting the blocks for several combinations of three conditions involving color, shape and size, and the use of NOT, students are asked to predict in advance how many blocks will satisfy each set of three conditions prior to sorting the blocks. The eight cases that must be considered can be represented in a Venn diagram as shown. The Roman numerals indicate these cases. In I are those NOT Red AND NOT Circle AND NOT Large. In II are those NOT Red AND Circle AND NOT Large. In III are those Red AND Circle AND NOT LARGE. Many students have difficulty seeing that NOT(NOT Red) is Red, and several examples are needed to have them become comfortable with this idea.



A systematic way to analyze how many possible values of each attribute are possible for each combination of conditions is developed. Use of the following table is encouraged.

COMBINATION OF CONDITIONS (As related to the Venn Diagram)	POSSIBLE NUMBER OF ATTRIBUTE VALUES			TOTAL
	SHAPE	COLOR	SIZE	
I				
II				
III				
IV				
V				
VI				
VII				
VIII				

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The use of the multiplication principle to determine how many blocks will satisfy each of the eight combinations of conditions is pointed out, and why it should be used is emphasized. How logical AND is related to multiplication, and how logical exclusive OR is related to addition is also emphasized while relating this kind of situation to combinatorial reasoning and ordering. Students must also realize that NOT has meaning only with relation to some defined universe and to recognized sets of values of the identified attributes or variables. They must also see that the total population of the universe is the same as the sum of the numbers that satisfy the different combinations of conditions. All of these ideas are generalized from considering the values of the attributes that these materials have to more abstract sets of attributes and values. Problems are worked that involve the options available when buying items like furniture and automobiles when certain conditions are imposed, or requirements must be met. This kind of thinking is needed in quality control of products being manufacured and in other commonplace problems. The efficacy of keeping the possibilities organized in table form is pointed out in these cases.

How classification related reasoning is applied when developing multiple relationships between values of different variables is considered using Mindbender problems that are available from Midwest Publications. The use of tables to array all potential two way relationships is encouraged. The conditions given can then be used to exclude those possibilities that do not meet the conditions, or that contradict required conditions. This provides a good opportunity to review contradicting conditions as these occurred in working with the attribute materials earlier. An example would be requiring a member of a class to be red and green simultaneously or square and circle simultaneously.

Students are shown how to first develop two way relationships and then to combine these into more extensive multiple relationships. The application of this kind of thinking in science and economics is emphasized. Students are encouraged to identify the variables that are involved and the values that these have, and then to systematically analyze the possible relationships using charts and tables.

Linear sequencing and working with linear patterns are introduced with patterns to be completed that are presented on the overhead projector. These involve numerical patterns, letter patterns, shape patterns and patterns of transformation. Students are given a lengthy assignment that requires them to extend several given patterns, and to insert missing elements into patterns. Students are reminded of the cases where pattern identification and analysis has been needed in solving problems given earlier in the course, and are encouraged to look for patterns as a basic approach to any problem solving or analysis activity.

Although the fundamental role that comparison plays in our thinking is constantly pointed out in the course, special comparison activities are introduced at this time. Geoblocks are a set of wooden blocks consisting of several different sizes and shapes of triangular prisms, rectangular solids, and a pyramid. These are related to each other in very specific ways. Students are introduced to the properties of these by having them sort them in various ways, and then having them build shapes like larger pieces in the set from two of the same smaller blocks,then from two different smaller blocks, then from three of the same smaller blocks, then from three different smaller blocks, etc. This gives them an opportunity to identify which blocks have the same volume, which have a

volume half that of some other block, which have a volume twice that of some block, one-third that of some block, one-fourth that of some block, etc. Each student is then given a set of three blocks that differ from each other and told to order these by volume. The students within each group check the orderings of the others in the group, and when all have been convinced that the orderings are correct, particular attention is paid to those cases where two blocks are in the same position in the order because they are equal in volume. Each student is then given two additional blocks, each different from any in that student's ordering, and told to insert these into the existing ordering. In this activity, the only way of establishing the ordering is by comparison, both direct and indirect. The students are free to choose some other block in the set against which two or more of their blocks can be compared in order to place them in the ordering. At this point it is usually necessary to point out such basic relationships as doubles of equals are equal, halves of equals are equal, etc. Once these orderings are settled upon, the students are asked to order this same set of five blocks according to the total surface area. This usually produces consternation on the part of some who make the assumption that this ordering is the same as the volume ordering. Another activity is to have a collection of 10 to 12 students order themselves along a wall according to their heights. This dramatically emphasizes how ordering is accomplished by using two way comparisons and then transitivity, and ordering two things often requires the use of a third element against which both can be compared. Problems are then assigned that involve ordering on several measurable characteristics such as area, length, depth, volume, degree of occurrence or performance on a test.

Rates of change in variables are introduced by using an activity called

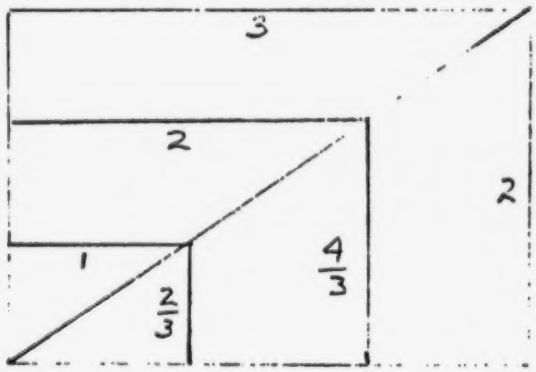
"Guess My Rule" to obtain values of a dependent variable that are associated with particular values of an independent variable. In this activity, the instructor has in mind some rule relating the dependent variable to the independent variable. Easy rules such as "multiply by three", "add four", or "square the number" are used to familiarize students with the process. Students are to input small whole numbers less than ten and the instructor responds to each with an output, using the rule. Students are to consider the pairs of values that result in order to determine the rule. A student who has discovered the rule is to test whether the discovery is correct by asking"if I give you ___, will you give me ____ in return?", not by blurting out the rule description. The tables of values resulting from each rule are written on the chalkboard and discussed. Often it is necessary to point out the importance of ordering the values of the I(ndependent) variable to see what pattern is developing in the values of the D(ependent) variable. This becomes necessary when a rule like "multiply by three and add five" is introduced. Determining the form of this rule requires recognizing a constant change in D for a given change in I. Students are encouraged to look for these successive differences in D and to place these in another column in the table. A CONSTANT change in D for a given change in I is characterized by rules that are of the form, $D = rI +/- k$, where r is the CONSTANT change in D and k is some starting point identified with a 0 value for I. The properties of this kind of rate of change, such as graphing as a straight line and proportional comparisons of change in D with change in I, are explored. The host of examples of this kind of constant rate of change or constant rate of comparison that we encounter in situations such as those involving distance-time, percent, density, and total cost are considered. The commonality of these is emphasized. Emphasis is also placed on those situations where constant rates

this common side. The common length is defined to be a unit of length for the activity with these blocks. Each student is given a large number of pieces of a particular kind. No student initially is given the squares (because working with them is too simple) or the hexagons. All students are asked to complete a table like that below by building each successively larger shape similar to the shape given to each student.

LENGTH OF SIDE OF THE SHAPE	NUMBER OF PIECES USED	AREA
1		
2		
3		
4		
5		

When all students have completed this table, the area of the major face of each different piece is defined to be a unit of area for all shapes made using that piece. Then the number of pieces also becomes a measure of the area of the shape made. Students are asked to predict how many pieces would be used, and hence what the area would be, when shapes are made with sides 6,7,etc. Recognition that growth in area is the square of the growth in a side length is generalized to include the changes in any corresponding linear measure such as circumference, diameter, etc. Thus if l is a side and a is the area of the smaller, and L the side and A the area of the larger, if $L = r\ l$, then $A = r^2\ a$. Problems that involve an area change for a given change in diameter, circumferences, radii, or sides of rectangles and squares are then worked within the groups and assigned

for outside work. Special attention is paid to the case of similar rectangles, since the ratio of the sides is a constant and there is a common diagonal line.



This can be related to the case of constant comparison and the straight line graph of the linear relationship first developed using Guess My Rule. Several problems are assigned for group work that require the students to use a comparison of areas in order to compare volumes, or vice versa, or to use a comparison of linear dimensions to compare areas or volumes. As a culminating activity, students solve the Nine Men in a Boat problem and the Tower of Hanoi and generate rules for finding the relationships relating the independent and dependent variables for the "n"th case. The latter is used to provide opportunity to discuss situations where binary decision making is used and hence the importance of the exponential function with base two. Examples are given from electricity, plumbing and computer design.

Analogies are introduced with several developmental activities. Overhead transparencies have been made of several of the pages in the Analogies series published by Midwest Publications. Students are given experience in identifying the relationships in analogies, in writing the relationship sentences that describe each pair of related words in an analogy and the analogy, in completing analogies and in creating analogies. They are made familiar with the more than

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sixty different relationships that can be represented in verbal analogy form. Students take one of the practice tests from Preparation for the Miller Analogies Test, Gruber and Gruber, and the results are gone over carefully. This work with verbal analogies provides an opportunity to stress the importance of good language use and of acquiring both general vocabulary and the vocabulary specific to knowledge areas. Figural analogies are also studied, since these give students what has proved to be much needed experience with spatial relations and the transformation of space forms and shapes.

Suggestions for general approaches to problem solving that are based on the work of Polya(1968), Osborn(1953) and deBono(1969,1970), among others, are given to the students. These are related to the activities of the course, and the cases where diagrams, charts, tables, and pictures have been used as aids during the work in the course are reviewed.

The class originally was designed with measurement as a final topic. It has never been possible to do the planned work with measurement theory and application. If the credit given for the course is again increased to five, this might be possible. Although initially planned to help freshmen and sophomores become better equipped to meet the thinking demands of various content courses on the campus, this course now enrolls significant numbers of juniors and seniors. Many of the academic advisors for the majors in the sciences, in economics, and the school of medicine recommend the course to their students, even at the upper division stages of their progress. The course has not yet been used to do a controlled study of how effective it is in meeting the goals. Several instruments have been studied and some piloted for this purpose, but most do not measure what the course is intended to achieve. During the fall of 1984, an instrument developed by C. Tomlinson-Keasey and Thomas Campbell will be used in

such a study. Three items have been added to this test to make it more accurately reflect the content of this particular course.

This course was designed to provide experiences with methods and materials diverse enough to appeal to the needs of the different conceptual styles that the students have. Some students respond well to verbal presentations while others need more graphic and pictorial modes. Responses of students in this class indicate that even college students need concrete experences to initiate them to unfamiliar material. If the manipulative materials are abandoned in favor of more symbolic and abstract methods prematurely, students will ask to have "the blocks brought back." The general approach of first having concrete situations with language and written symbols, then carefully representing these with language and symbolic description, and finally dealing with more abstract situations, even for college students, has been justified by the experience in this class. One of the most significant outcomes of teaching this class for several years has been a recognition of the need for a major overhauling of the secondary school curriculum. The predominant goals of memorization must be replaced by goals related to thinking and problem solving. Students must come to college with the prevailing orientation toward searching for memorized formulas and procedures replaced by one that leads them to think their way through problems.

SAMPLE PROBLEMS

LOGICAL REASONING

In Shakespeare's Merchant of Venice, Portia presents her suitor with three chests; one of lead, one of gold, and one of silver. One contains her portrait, and if her suitor can determine which it is, Portia will marry him. Portia never lies and tells him that at most one inscription is true. The inscriptions are:

GOLD: The portrait is in this chest.

SILVER: The portrait is not in this chest.

LEAD: The portrait is not in the gold chest.

Which chest should the suitor choose?

Why? Validate your conclusion using truth tables.

COMBINATORIAL REASONING

A committee of six men is to be chosen from a group of twelve men, six of whom are Republicans and six of whom are Democrats. In how many different ways might this committee be constituted with at least one Democrat on it?

PATTERNS

On a panel are 100 switches numbered 1 - 100. All are closed. The first passerby changes each switch to the opposite state. The second passerby changes every second switch to the opposite state; the third passerby changes each third switch to the opposite state,etc. until 100 people have come by. After the last person has passed which switches are open? (Hint: There are ten of them)

ORDERING

X = 2/3 Y; Y = 3/5 Z; W = 1/2 Z; U = 5/4 V; and V = 3/4 Y.

Given these several two-way comparisons between the quantities represented by X,Y,Z,W,U,V, order the quantities from the least to the greatest.

PROPORTIONAL REASONING

If it takes one pint of paint to paint a statue that is five feet high, how many similar statues only one foot high can be painted to the same thickness with ten pints of paint?

CLASSIFICATION

At Ajax plastics, the shipping clerk, the stockgirl, the saleswoman and the cashier are named Rose, Hannah, Geraldine and Maryjo, but Mr. Bigwig, the President of Ajax, can't remember which person has which name. Bigwig does, however, know the following facts about them.

Hannah likes both the saleswoman and the cashier.

Maryjo rides to work with the saleswoman and the cashier.

The shipping clerk comes to work alone.

Rose is slightly jealous of the cashier.

Match the women with their jobs at Ajax.

CLASSIFICATION

An auto dealer stocks eight colors - red, blue, bronze, silver, green, black, yellow and brown. The models stocked include 4 door hatchbacks, 2 door hatchbacks, 4 door sedans, and station wagons. The transmissions available are automatics, 4 speed manuals, 5 speed manuals, and 4 speed with overdrive. Both 4 cylinder and V6 engines are available. The dealer keeps on hand one of each combination of color, body style, engine and transmission that are available.

1 If I go to the dealer having decided on a 4 door sedan, how many cars do I have to choose from?

2. If I additionally decide on an automatic transmission, how many cars might I choose from?

3. If the dealer parks all of the brown cars on a separate lot, how many cars on that lot?

4. How many hatchbacks with 4 speed transmissions are in the dealer's stock?

RATES OF VARIATION

The following value of a dependent variable B were found for the given values of the independent variable A.

VARIABLE A	3	5	0	4	6
VARIABLE B	32	74	-1	51	101

Write an expression that shows how a value of B can be found from a given value of A.

ANALOGIES

ABNORMAL			
ABSENT			
APPROXIMATE			
AUDIO			
SUGAR			

Fill in the cells in the table above with words from the lists below so that each row is an analogy. The words for each column must be taken from the list for that column and each word may be used only once.

Column 2; hear, lemon, normal, precise, present

Column 3: almost, away, sweet, unusual, visual

Column 4: at hand, exactly, see, sour, usual

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